

Solutions to Algebraic Expressions – Test A

Fundamentals ✓ Challenge Expert

1. a) $(11+x)(3-x) = 33 - 11x + 3x - x^2 = 33 - 8x - x^2 \text{ A1}$
- b) $(x+5)(x^2 - 8x + 6) = x^3 - 8x^2 + 6x + 5x^2 - 40x + 30 \text{ M1}$
 $= x^3 - 3x^2 - 34x + 30 \text{ A1}$
- c) $(x-2)(x+3)(x-5) = (x^2 + x - 6)(x-5) \text{ M1} \quad \leftarrow$
 $= x^3 - 5x^2 + x^2 - 5x - 6x + 30 \text{ M1}$
 $= x^3 - 4x^2 - 11x + 30 \text{ A1}$

Alternative Method: You could instead multiply the second pair of brackets first to get
 $(x-2)(x^2 - 2x - 15)$

[6 Marks]

2. a) $36 - 9x - x^2 = -(x^2 + 9x - 36) = -(x+12)(x-3) \text{ M1}$
 $= (x+12)(3-x) \text{ A1}$
- b) $2y^2 - y - 21 = (2y-7)(y+3) \text{ A1A1}$
- c) $2x^3 - 4x^2 + 2x = 2x(x^2 - 2x + 1) \text{ M1}$
 $= 2x(x-1)^2 \text{ A1}$

[6 Marks]

3. a) $\frac{7y^{11}}{35y^6} = \frac{1}{5} \times \frac{y^{11}}{y^6} = \frac{1}{5} \times y^{11-6} = \frac{y^5}{5} \text{ A1}$
- b) $5x^2 \div x^{-4} = 5x^{2-(-4)} = 5x^6 \text{ A1}$
- c) $(x^5)^{\frac{3}{5}} = x^{\frac{5 \times 3}{5}} = x^3 \text{ A1}$

[3 Marks]

4. a) $\sqrt{75} + \sqrt{27} = \sqrt{25 \times 3} + \sqrt{9 \times 3} = \sqrt{25} \times \sqrt{3} + \sqrt{9} \times \sqrt{3} \text{ M1}$
 $= 5\sqrt{3} + 3\sqrt{3} = 8\sqrt{3} \text{ A1}$
- b) $\frac{\sqrt{60}}{\sqrt{3}} = \sqrt{\frac{60}{3}} = \sqrt{20} \text{ M1}$
 $= \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} \text{ M1}$
 $= 2\sqrt{5} \text{ A1}$
- c) $\sqrt{2} \times 3\sqrt{8} = 3\sqrt{2 \times 8} \text{ M1}$
 $= 3\sqrt{16} = 3 \times 4 = 12 \text{ A1}$

[7 marks]

5. a) $\sqrt{x^4} \times \sqrt{x^7} = (x^4)^{\frac{1}{2}} \times (x^7)^{\frac{1}{2}} \text{ M1}$
 $= x^{\frac{4 \times 1}{2}} \times x^{\frac{7 \times 1}{2}} = x^2 \times x^{\frac{7}{2}} \text{ M1}$
 $= x^{\frac{2+7}{2}} = x^{\frac{11}{2}} \text{ A1}$
- b) $(\sqrt[3]{x})^5 = \left(x^{\frac{1}{3}}\right)^5 \text{ M1}$
 $= x^{\frac{1 \times 5}{3}} = x^{\frac{5}{3}} \text{ A1}$
- c) $(\sqrt{x})^{-\frac{3}{2}} = \left(x^{\frac{1}{2}}\right)^{-\frac{3}{2}} \text{ M1}$
 $= x^{\frac{1 \times -\frac{3}{2}}{2}} = x^{-\frac{3}{4}} \text{ A1}$

[7 Marks]

6. a) $8^{\frac{1}{3}} = \sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2 \text{ A1}$
- b) $16^{-\frac{3}{2}} = (\sqrt{16})^{-3} \text{ M1}$
 $= 4^{-3} = \frac{1}{4^3} = \frac{1}{64} \text{ A1}$

[3 Marks]

$$\begin{aligned}
 7. \quad & \frac{1}{-1+2\sqrt{2}} = \frac{1}{-1+2\sqrt{2}} \times \frac{-1-2\sqrt{2}}{-1-2\sqrt{2}} = \frac{-1-2\sqrt{2}}{(-1+2\sqrt{2})(-1-2\sqrt{2})} \quad \text{M1} \\
 & = \frac{-1-2\sqrt{2}}{(-1)^2 - (2\sqrt{2})^2} \quad \text{M1} \\
 & = \frac{-1-2\sqrt{2}}{1-8} \quad \text{M1} \\
 & = \frac{-1-2\sqrt{2}}{-7} = \frac{1+2\sqrt{2}}{7} \quad \text{A1}
 \end{aligned}$$

Technique: Use the difference of two squares technique to eliminate surds from the denominator by multiplying by a fraction equivalent to 1

[4 Marks]

$$\begin{aligned}
 8. \quad \text{a)} \quad & 7y^3 \times 4y^5 \times \frac{1}{2}y^2 = \left(7 \times 4 \times \frac{1}{2}\right) \times (y^3 \times y^5 \times y^2) \quad \text{M1} \\
 & = 14 \times y^{3+5+2} = 14y^{10} \quad \text{A1}
 \end{aligned}$$

$$\begin{aligned}
 \text{b)} \quad & (8x^6)^{-\frac{2}{3}} = 8^{-\frac{2}{3}} \times (x^6)^{-\frac{2}{3}} \quad \text{M1} \\
 & = (\sqrt[3]{8})^{-2} \times x^{6 \times -\frac{2}{3}} = 2^{-2} \times x^{-4} \quad \text{M1}
 \end{aligned}$$

$$\begin{aligned}
 \text{c)} \quad & \sqrt{45} - \sqrt{44} - \sqrt{20} = \sqrt{9 \times 5} - \sqrt{4 \times 11} - \sqrt{4 \times 5} = \sqrt{9} \times \sqrt{5} - \sqrt{4} \times \sqrt{11} - \sqrt{4} \times \sqrt{5} \quad \text{M1} \\
 & = 3\sqrt{5} - 2\sqrt{11} - 2\sqrt{5} \quad \text{M1} \\
 & = \sqrt{5} - 2\sqrt{11} \quad \text{A1}
 \end{aligned}$$

[8 Marks]

$$\begin{aligned}
 9. \quad & x^2 - 4y^2 = x^2 - (2y)^2 \quad \text{M1} \quad \blacktriangleleft \\
 & = (x+2y)(x-2y) \quad \text{A1}
 \end{aligned}$$

Technique: Use the difference of two squares:
 $a^2 - b^2 = (a+b)(a-b)$

[2 Marks]

$$\begin{aligned}
 10. \quad \text{a)} \quad & (2+\sqrt{5})(4-\sqrt{5}) = 8 - 2\sqrt{5} + 4\sqrt{5} - (\sqrt{5})^2 \quad \text{M1} \\
 & = 8 - 2\sqrt{5} + 4\sqrt{5} - 5 = 3 + 2\sqrt{5} \quad \text{A1}
 \end{aligned}$$

$$\begin{aligned}
 \text{b)} \quad & \frac{3}{(2+\sqrt{5})(4-\sqrt{5})} = \frac{3}{3+2\sqrt{5}} = \frac{3}{3+2\sqrt{5}} \times \frac{3-2\sqrt{5}}{3-2\sqrt{5}} \quad \text{M1} \\
 & = \frac{3(3-2\sqrt{5})}{3^2 - (2\sqrt{5})^2} \quad \text{M1} \\
 & = \frac{9-6\sqrt{5}}{9-20} = \frac{9-6\sqrt{5}}{-11} \quad \text{M1} \\
 & = -\frac{9}{11} + \frac{6}{11}\sqrt{5} \quad \text{A1}
 \end{aligned}$$

[6 Marks]

$$\begin{aligned}
 11. \quad \text{a)} \quad & (9x^2)^{\frac{3}{2}} = 9^{\frac{3}{2}} \times (x^2)^{\frac{3}{2}} = (\sqrt{9})^3 \times x^{2 \times \frac{3}{2}} \quad \text{M1} \\
 & = 3^3 \times x^3 = 27x^3 \quad \text{A1}
 \end{aligned}$$

$$\begin{aligned}
 \text{b)} \quad & \sqrt{x^7} \times \sqrt{25x^4} = (x^7)^{\frac{1}{2}} \times \sqrt{25} \times \sqrt{x^4} = (x^7)^{\frac{1}{2}} \times 5 \times (x^4)^{\frac{1}{2}} \quad \text{M1} \\
 & = 5 \times x^{\frac{7}{2}} \times x^{\frac{4}{2}} \quad \text{M1} \\
 & = 5 \times x^{\frac{7}{2}+2} = 5x^{\frac{11}{2}} \quad \text{A1}
 \end{aligned}$$

[5 Marks]

$$\begin{aligned}
 12. \quad & 27^x = 9 \therefore (3^3)^x = 3^2 \quad \text{M1} \quad \blacktriangleleft \\
 & \therefore 3^{3x} = 3^2 \therefore 3x = 2 \quad \text{M1}
 \end{aligned}$$

Technique: Write 27 and 9 as powers of 3

$$\begin{aligned}
 & \therefore x = \frac{2}{3} \quad \text{A1}
 \end{aligned}$$

[3 Marks]

TOTAL 60 MARKS