

A level Mathematics Year 11 to 12 transition

### 1

**Contents**

1. Introduction
2. [Expanding brackets and simplifying expressions](#_Expanding_brackets_and) \*
3. [Factorising expressions](#_Factorising_expressions) \*
4. [Rules of indices](#_Rules_of_indices) \*
5. [Surds](#_Surds) \*
6. [Rearranging equations](#_Rearranging_equations) \*
7. [Completing the square](#_Completing_the_square) \*
8. [Solving quadratic equations by factorisation](#_Solving_quadratic_equations) \*
9. [Solving quadratic equations by completing the square](#_Solving_quadratic_equations_1) \*
10. [Solving quadratic equations by using the formula](#_Solving_quadratic_equations_2) \*
11. [Solving linear simultaneous equations by elimination](#_Solving_linear_simultaneous) \*
12. [Solving linear simultaneous equations by substitution](#_Solving_linear_simultaneous_1) \*
13. [Solving linear and quadratic simultaneous equations](#_Solving_linear_and) \*
14. [Solving simultaneous equations graphically](#_Solving_simultaneous_equations)
15. [Linear inequalities](#_Linear_inequalities) \*
16. [Quadratic inequalities](#_Quadratic_inequalities)
17. [Sketching cubic and reciprocal graphs](#_Sketching_cubic_and)
18. [Translating graphs](#_Translating_graphs)
19. [Stretching graphs](#_Stretching_graphs)
20. [Straight line graphs](#_Straight_line_graphs)
21. [Parallel and perpendicular lines](#_Parallel_and_perpendicular)
22. [Volume and surface area of 3D solids](#_Volume_and_surface)
23. [Pythagoras’ theorem](#_Pythagoras’_theorem) \*
24. [Trigonometry in right-angled triangles](#_Trigonometry_in_right-angled) \*
25. [The cosine rule](#_The_cosine_rule)
26. [The sine rule](#_The_sine_rule)
27. [Area of a triangle using ½absin C](#_Area_of_a)

2

## Expanding brackets and simplifying expressions

### Answers

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **1** | **a****c** | 6*x* – 3–3*xy* + 2*y*2 | **b** | –10*pq* – 8*q*2 |
| **2** | **a** | 21*x* + 35 + 12*x* – 48 = 33*x* – 13 |  |
|  | **b** | 40*p* – 16 – 12*p* – 27 = 28*p* – 43 |  |
|  | **c** | 27*s* + 9 – 30*s* + 50 = –3*s* + 59 = 59 – 3*s* |  |
|  | **d** | 8*x* – 6 – 3*x* – 5 = 5*x* – 11 |  |
| **3** | **a c** | 12*x*2 + 24*x* **b**10*h* – 12*h*3 – 22*h*2 **d** | 20*k*3 – 48*k*21*s*2 – 21*s*3 – 6*s* |
| **4** | **a c** | –*y*2 – 4 **b**2*p* – 7*p*2 **d** | 5*x*2 – 11*x*6*b*2 |
| **5** | *y* – 4 |  |  |
| **6** | **a** | –1 – 2*m* **b** | 5*p*3 + 12*p*2 + 27*p* |

**7** 7*x*(3*x* – 5) = 21*x*2 – 35*x*

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **8** | **a** | *x*2 + 9*x* + 20 | **b** | *x*2 + 10*x* + 21 |
|  | **c** | *x*2 + 5*x* – 14 | **d** | *x*2 – 25 |
|  | **e** | 2*x*2 + *x* – 3 | **f** | 6*x*2 – *x* – 2 |
|  | **g** | 10*x*2 – 31*x* + 15 | **h** | 12*x*2 + 13*x* – 14 |
|  | **i** | 18*x*2 + 39*xy* + 20*y*2 | **j** | *x*2 + 10*x* + 25 |
|  | **k** | 4*x*2 − 28*x* + 49 | **l** | 16*x*2 − 24*xy* + 9*y*2 |

**9** 2*x*2 − 2*x* + 25

**10 a**

*x*2 1 2

*x*2

**b** *x*2  2  1

*x*2

## Factorising expressions

### Answers

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **1** | **a c** | 2*x*3*y*3(3*x* – 5*y*) 5*x*2*y*2(5 – 2*x* + 3*y*) | **b** | 7*a*3*b*2(3*b*3 + 5*a*2) |
| **2** | **a c e g** | (*x* + 3)(*x* + 4) (*x* – 5)(*x* – 6)(*x* – 9)(*x* + 2)(*x* – 8)(*x* + 5) | **b d f h** | (*x* + 7)(*x* – 2)(*x* – 8)(*x* + 3)(*x* + 5)(*x* – 4)(*x* + 7)(*x* – 4) |
| **3** | **a c** | (6*x* – 7*y*)(6*x* + 7*y*)2(3*a* – 10*bc*)(3*a* + 10*bc*) | **b** | (2*x* – 9*y*)(2*x* + 9*y*) |
| **4** | **a****c e** | (*x* – 1)(2*x* + 3)(2*x* + 1)(*x* + 3)(5*x* + 3)(2*x* +3) | **b****d f** | (3*x* + 1)(2*x* + 5) (3*x* – 1)(3*x* – 4)2(3*x* – 2)(2*x* –5) |

**5 a** 2(*x*  2)

*x* 1

*x*

**b** *x* 1

**c** *x*  2

*x*

**e** *x*  3

*x*

*x*

**d** *x*  5

*x*

**f** *x*  5

**6 a** 3*x*  4

*x*  7

**c** 2  5*x* 2*x*  3

**b** 2*x*  3

3*x*  2

1. 3*x*  1

*x*  4

**7** (*x* + 5)

**8** 4(*x*  2)

*x*  2

## Rules of indices

### Answers

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **1** | **a** | 1 | **b** | 1 | **c** | 1 | **d** | 1 |
| **2** | **a** | 7 | **b** | 4 | **c** | 5 | **d** | 2 |
| **3** | **a** | 125 | **b** | 32 | **c** | 343 | **d** | 8 |

1. **a** 1 **b** 1

25 64

**c** 1 **d** 1

32 36

3*x*3

1. **a**

2

**b** 5*x*2

**c** 3*x* **d**

1

*y*

2*x*2

1. *y* 2
2. *c*–3
3. 2*x*6 **h** *x*
4. **a** 1 **b** 1

2 9

**d** 1 **e** 4

4 3

**c** 8

3

**f** 16

9

1

1. **a** *x*–1 **b** *x*–7 **c** *x*4

2

**d** *x*5

1. **a** 1

*x*3

**d**

5 *x*2

1

1. *x* 3

 2

1. *x* 3

5 *x*

1

4 *x*3

|  |  |  |  |
| --- | --- | --- | --- |
| **b** | 1 |  | **c** |
| **e** |  | 1*x* | **f** |

1

1. **a** 5*x* 2

 1

**b** 2*x*–3 **c**

1

1 *x*4

3

1. 2*x* 2
2. 4*x* 3
3. 3*x*0
4. **a**

*x*3  *x*2

**b** *x*3  *x*

**c** *x*2  *x*7

## Surds

### Answers

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **1** | **a** | 3 5 | **b** | 5 | 5 |
|  | **c** | 4 3 | **d** | 5 | 7 |
|  | **e** | 10 3 | **f** | 2 | 7 |
|  | **g** | 6 2 | **h** | 9 | 2 |
| **2** | **a** | 15 2 | **b** | 5 |
|  | **c** | 3 2 | **d** | 3 |
|  | **e** | 6 7 | **f** | 5 3 |

**3 a** −1 **b** 9 

3

**c** 10 5 7 **d** 26  4

2

1. **a** 5

5

**c** 2 7

7

**e**

2

**g** 3

3

**b** 11

11

**d** 2

2

**f**

5

**h** 1

3

1. **a** 3 5

4

**b** 2(4  3)

13

**c** 6(5 2)

23

1. *x* − *y*

*x*  *y*

1. **a** 3  2

2

**b** *x*  *y*

## Rearranging equations

### Answers

1. *d* = *C*

**

1. *w*  *P*  2*l*

2

1. *T*  *S D*
2. *t*  *q*  *r p*
3. *t* 

2*u* 2*a* 1

1. *x* 

*V*

*a*  4

**7** *y* = 2 + 3*x* **8**

*a*  3*x*  1

*x*  2

**9** *d*  *b*  *c x*

**10** *g*  2*h*  9

1.  *h*

*A*

**

**11** *e* 

1

*x*  7

**12** *x*  4 *y*  3

2  *y*

3*V*

3

4**

1. **a**

3*V*

2** *h*

*r*  **b** *r* 

* 1. *r* 

*P*

**  2

* 1. *r* 
1. **a**

*x*  *abz*

*cdy*

**b** *x* 

3*dz* 4** *cpy*2

1. sin *B*  *b* sin *A*

*a*

1. cos *B* 

*a*2  *c*2  *b*2

2*ac*

1. **a**

*x*  *q*  *pt*

**b** *x*  3 *py*  2 *pqy*  *y*(3  2*q*)

*q*  *ps*

3 *p*  *apq* 3  *aq*

## Completing the square

### Answers

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **1** | **a** | (*x* + 2)2 – 1 | **b** | (*x* – 5)2 – 28 |
|  | **c** | (*x* – 4)2 – 16 | **d** | (*x* + 3)2 – 9 |

**e** (*x* – 1)2 + 6 **f**

 *x* 





3 2



2



 17

4

**2 a** 2(*x* – 2)2 – 24 **b** 4(*x* – 1)2 – 20

**c** 3(*x* + 2)2 – 21 **d**

2 *x* 





3 2



2



 25

2

 3 2 39  1 2 1

**3 a** 2 *x*   

4

  8

**b** 3 *x*   

  3

3

 3 2 9  5 2 11

**c** 5 *x*   

10

  20

**d** 3 *x*   

  12

6

**4** (5*x* + 3)2 + 3

## Solving quadratic equations by factorisation

Answers

1. **a** *x* = 0 or *x* =  2

3

**b** *x* = 0 or *x* = 3

4

|  |  |  |  |
| --- | --- | --- | --- |
| **c** | *x* = –5 or *x* = –2 | **d** | *x* = 2 or *x* = 3 |
| **e** | *x* = –1 or *x* = 4 | **f** | *x* = –5 or *x* = 2 |
| **g** | *x* = 4 or *x* = 6 | **h** | *x* = –6 or *x* = 6 |
| **i** | *x* = –7 or *x* = 4 | **j** | *x* = 3 |

**k** *x* =  1 2

or *x* = 4 **l** *x* =  2

3

or *x* = 5

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **2** | **a** | *x* = –2 or *x* = 5 | **b** | *x* = –1 or *x* = 3 |
|  | **c** | *x* = –8 or *x* = 3 | **d** | *x* = –6 or *x* = 7 |
|  | **e** | *x* = –5 or *x* = 5 | **f** | *x* = –4 or *x* = 7 |

1. *x* = –3 or *x* = 2 1

2

1. *x* =  1 3

or *x* = 2

## Solving quadratic equations by completing the square

Answers

**3 a** *x* = 2 +

7

**c** *x* = –4 +

6.5

or *x* = 2 –

or *x* = –4 –

7

21

**b** *x* = 5 +

**d** *x* = 1 +

or *x* = 5 – or *x* = 1 –

21

21

21

7

7

**e** *x* = –2 +

**4 a** *x* = 1 +

14

**c** *x* =

5  13

2

or *x* = –2 –

or *x* = 1 – or *x* =

6.5

14

5  13

2

**f** *x* =

**b** *x* =

3  89

10

3  23

2

or *x* =

or *x* =

3  89

10

3  23

2


## Solving quadratic equations by using the formula

### Answers

**5 a** *x* = –1 + 3

3

or *x* = –1 – 3

3

**b** *x* = 1 + 3 2

2

or *x* = 1 – 3 2

2

1. *x* =
2. *x* =

3 

20

or *x* =

89 or *x* =

7  41

2

7  41

2

3  89

20

1. **a** *x* =

7  17

8

**b** *x* = –1 +

**c** *x* = –1 2

3

or *x* =

or *x* = –1 – or *x* = 2

7  17

8

10

10

## Sketching quadratic graphs

### Answers

**1**



1. **a b c**



1. **a b c**



**d e f**

**4**



**5 a b c**



**6**



Line of symmetry at *x* = −1.

## Solving linear simultaneous equations by elimination

### Answers

**1** *x* = 1, *y* = 4

1. *x* = 3, *y* = –2
2. *x* = 2, *y* = –5
3. *x* = 3, *y* = – 1

2

1. *x* = 6, *y* = –1
2. *x* = –2, *y* = 5

## Solving linear simultaneous equations by substitution

### Answers

1. *x* = 9, *y* = 5
2. *x* = –2, *y* = –7
3. *x* = 1 , *y* = 3 1

2 2

1. *x* = 1 , *y* = 3

2

1. *x* = –4, *y* = 5
2. *x* = –2, *y* = –5
3. *x* = 1 , *y* = 1 3

4 4

1. *x* = –2, *y* = 2 1

2

1. *x* = –2 1 , *y* = 5 1

2 2

## Solving linear and quadratic simultaneous equations

Answers

**1** *x* = 1, *y* = 3

 

**2** *x* = 2, *y* = 4

 *x* = 4, *y* = 2

**3** *x* = 1, *y* = −2

 *x* = 2, *y* = –1

**4** *x* = 4, *y* = 1

 

**5** *x* = 3, *y* = 4

 *x* = 2, *y* = 1

**6** *x* = 7, *y* = 2

 *x* = −1, *y* = −6

**7** *x* = 0, *y* = 5

 *x* = –5, *y* = 0

**8** *x* = , *y* = 

 *x* = 3, *y* = 5

**9** *x* = –2, *y* = –4

 *x* = 2, *y* = 4

**10** *x* = , *y* = 6

 *x* = 3, *y* = 5

**11** *x* = , *y* = 

 *x* = , *y* = 

**12** *x* = , *y* =  *x* = , *y* = 

## Solving simultaneous equations graphically

### Answers

**1 a** *x* = 2, *y* = 5

**b** *x* = 2, *y* = −3

1. *x* = −0.5, *y* = 2.5

**2 a** *x* = −2, *y* = 2

**b** *x* = 0.5, *y* = 0.5

**c** *x* = −1, *y* = −2

**3 a** *x* = 1, *y* = 0 and *x* = 4, *y* = 3

**b** *x* = −2, *y* = 7 and *x* = 2, *y* = −5

**c** *x* = −2, *y* = 5 and *x* = −1, *y* = 4

1. *x* = −3, *y* = 4 and *x* = 4, *y* = −3
2. **a i** *x* = 2.5, *y* = −2 and *x* = −0.5, *y* = 4

**ii** *x* = 2.41, *y* = −1.83 and *x* = −0.41, *y* = 3.83

**b** Solving algebraically gives the more accurate solutions as the solutions from the graph are only estimates, based on the accuracy of your graph.

## Linear inequalities

### Answers

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **1** | **a****d** | *x* > 4*x* > – 7 | **b****e** | *x* ≤ 2*x* ≥ 10 | **c****f** | *x* ≤ –1*x* < –15 |
| **2** | **a** | 2*x* < –20 | **b** | *x* ≤ 3.5 | **c** | *x* < 4 |
| **3** | **a** | *x* ≤ –4 | **b** | –1 ≤ *x* < 5 | **c** | *x* ≤ 1 |
|  | **d** | *x* < –3 | **e** | *x* > 2 | **f** | *x* ≤ –6 |

1. **a** *t* < 5

2

**b** *n* ≥ 7 5

1. **a** *x* < –6 **b** *x* < 3

2

1. *x* > 5 (which also satisfies *x* > 3)

## Quadratic inequalities

### Answers

**1** –7 ≤ *x* ≤ 4

**2** *x* ≤ –2 or *x* ≥ 6

**3** 1  *x*  3

2

**4** *x* <  3

2

or *x* > 1

2

**5** –3 ≤ *x* ≤ 4

**6** –3 ≤ *x* ≤ 2

1. 2 < *x* < 2 1

2

1. *x*  3

2

or *x*  5

3

## Sketching cubic and reciprocal graphs

### Answers

1. **a** i – C
2. – E
3. – B
4. – A
5. – F
6. – D

**b ii iv**

**vi**

**2 3**



**4 5**

**6 7**

**8 9**

***y***

***O***

***x***

**10 11**

## Translating graphs

### Answers

### **1 2**

**3**



**4** *C*1: *y* = f(*x* – 90°)

*C*2: *y* = f(*x*) – 2

**5** *C*1: *y* = f(*x* – 5)

*C*2: *y* = f(*x*) – 3

1. **a b**



## Stretching graphs

### Answers

1. **a b**

**8 9**



**10**



**11** *y* = f(2*x*)

**12** *y* = –2f(2*x*) or *y* = 2f(–2*x*)

1. **a b**



**14**



**15**



## Straight line graphs

## Answers

**1 a** *m* = 3, *c* = 5 **b** *m* =  1 , *c* = –7

2

**c** *m* = 2, *c* =  3

2

**e** *m* = 2 , *c* =  7 or –2 1

1. *m* = –1, *c* = 5

**f** *m* = –5, *c* = 4

3 3 3

**2**

|  |  |  |
| --- | --- | --- |
| **Gradient** | ***y*-intercept** | **Equation of the line** |
| 5 | 0 | *y* = 5*x* |
| –3 | 2 | *y* = –3*x* + 2 |
| 4 | –7 | *y* = 4*x* –7 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **3** | **a** | *x* + 2*y* + 14 = 0 | **b** | 2*x* – *y* = 0 |
|  | **c** | 2*x* – 3*y* + 12 = 0 | **d** | 6*x* + 5*y* + 10 = 0 |

**4** *y* = 4*x* – 3

**5** *y* =  2 *x* + 7

3

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **6** | **a** | *y* = 2*x* – 3 | **b** | *y* =  1 *x* + 62 |
|  | **c** | *y* = 5*x* –2 | **d** | *y* = –3*x* + 19 |

**7** *y*   3 *x*  3 , the gradient is  3

and the *y*-intercept is 3.

2 2

The line intercepts the axes at (0, 3) and (2, 0).

Students may sketch the line or give coordinates that lie on the line such as 1,



3  or 4, 3.

2



 

## Parallel and perpendicular lines

### Answers

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **1** | **a** | *y* = 3*x* –7 | **b** | *y* = –2*x* + 5 |
|  | **c** | *y* = – 1 *x* | **d** | *y* = 3 *x* + 8 |

2 2

1. *y* = −2*x* – 7

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **3** | **a** | *y* = – 1 *x* + 2 | **b** | *y* = 3*x* + 7 |
|  | **c** | *y* = –4*x* + 35 | **d** | *y* = 5 *x* – 8 |
|  |  |  |  | 2 |
| **4** | **a** | *y* = – 1 *x* | **b** | *y* = 2*x* |
| **5** | **a** | Parallel | **b** | Neither | **c** | Perpendicular |
|  | **d** | Perpendicular | **e** | Neither | **f** | Parallel |
| **6** | **a** | *x* + 2*y* – 4 = 0 | **b** | *x* + 2*y* + 2 = 0 | **c** | *y* = 2*x* |

2

2

## Volume and surface area of 3D solids

### Answers

**1 a** *V* = 396 cm3 **b** *V* = 75 000 cm3

**c** *V* = 402.5 cm3 **d** *V* = 200*π* cm3

**e** *V* = 1008*π* cm3 **f** *V=* 1372 *π* cm3

3

**g** *V* = 121.5*π* cm3 **h** *V* = 18*π* cm3

**i** *V* = 48*π* cm3 **j** *V* = 98 *π* cm3

3

1. 17 cm
2. 17 cm
3. *V* = *x*3 + 17 *x*2 + 4*x*

2

1. 60 cm3
2. 21.4 cm
3. 32 : 9
4. *r*  3 36*x*

## Pythagoras’ theorem

### Answers

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **1** | **a****c** | 10.3 cm58.6 mm | **b****d** | 7.07 cm8.94 cm |
| **2** | **a** | 4 3 cm | **b** | 2 21 cm |
|  | **c** | 8 17 mm | **d** |  18 5 mm |
| **3** | **a** | 18 13 mm | **b** | 2 145 mm |
|  | **c** | 42 2 mm | **d** | 6 89 mm |
| **4** | 95.3 | mm |  |  |
| **5** | 64.0 | km |  |  |
| **6** | 3 5 | units |  |  |
| **7** | 4 3 | cm |  |  |

## Trigonometry in right-angled triangles

### Answers

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **1** | **a** | 6.49 cm | **b** | 6.93 cm | **c** | 2.80 cm |  |
|  | **d** | 74.3 mm | **e** | 7.39 cm | **f** | 6.07 cm |  |  |
| **2** | **a** | 36.9° | **b** | 57.1° | **c** | 47.0° | **d** | 38.7° |

1. 5.71 cm
2. 20.4°
3. **a** 45° **b** 1 cm **c** 30° **d** cm

3

## The cosine rule

### Answers

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **6** | **a** | 6.46 cm | **b** | 9.26 cm | **c** | 70.8 mm | **d** | 9.70 cm |
| **7** | **a** | 22.2° | **b** | 52.9° | **c** | 122.9° | **d** | 93.6° |
| **8** | **a** | 13.7 cm | **b** | 76.0° |  |  |  |  |

## The sine rule

### Answers

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **9 a** | 4.33 cm | **b** | 15.0 cm | **c** | 45.2 mm | **d** | 6.39 cm |
| **10 a** | 42.8° | **b** | 52.8° | **c** | 53.6° | **d** | 28.2° |
| **11 a** | 8.13 cm | **b** | 32.3° |  |  |  |  |

## Area of a triangle using ½absinC

### Answers

**12 a** 18.1 cm2 **b** 18.7 cm2 **c** 693 mm2

**13** 5.10 cm

1. **a** 6.29 cm **b** 84.3° **c** 5.73 cm **d** 58.8°
2. 15.3 cm